PROBLEM OF THE SUPERSONIC FLOW AROUND A THIN FINITE SPAN WING WITH COMPLETELY SUBSONIC LEADING EDGES

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E. A. Krasil'shchikova [1, 2] solved the problem of the supersonic flow around a slightly cambered lifting surface of a finite span wing with subsonic edges under the condition of a supersonic section in the nose part of the leading edges. Starting from the condition that the velocity potential vanish on the part of the base plane outside the projection of the wing, the problem reduces to a two-dimensional Abel integral equation in the normal derivative of the potential outside the projection of the wing on the base plane. The inversion of the Abel integral is known. The solution is also known for the problem of the flow around a flat triangular wing with completely subsonic leading edges (conical flow) [3].

The method of replacement of the wing nose by a flat triangular plate with subsonic leading edges (assumption of conical flow in the nose part of the wing) or replacement of a nose with subsonic edges by some nose with sonic edges, which reduces the solution of the problem to the Krasil'shchikova algorithm, is ordinarily utilized in computations of the total aerodynamic characteristics of nonplanar wings with completely subsonic leading edges.

The problem of the flow around a nonplanar wing with completely subsonic leading edges was examined in [4] in the same formulation as in [1, 2] for a wing with partially supersonic leading edges (the potential in the base plane outside the wing projection is zero). The problem reduces to a two-dimensional Volterra-type integral equation in the velocity potential whose solution is possible by successive approximations. The zeroth approximation is given arbitrarily from some assumptions.

In this paper the solution of the problem of the flow around a slightly cambered wing with completely subsonic leading edges is based, exactly as in [1, 4], on the condition that the perturbation potential on the base plane outside the domain of wing projection equals zero. A Volterra-type integral equation of the second kind is obtained for the parameter governing the flow, the normal derivative of the potential on the base plane along one of the wing sides. The possibility is shown of solving this equation by successive approximations. The solution is a series whose terms are multiple integrals of known functions. The first term of the series (the zeroth approximation) reflects the main regularities of perturbation formation at the point under consideration. A comparison is presented between the governing parameter of the flow, evaluated in a zeroth approximation, and the known exact solution obtained by another method in the case of conical flow. The agreement is good in a broad range of leading edge deviations from the sonic. There is no practical necessity to perform the evaluation of the remaining terms of the series (multiple integrals). Finding the first term of the series consists of evaluating single and double integrals of known functions.

The gas dynamics equations can be reduced to the wave equation for the perturbation velocity potential for supersonic flow ($M > 1$) around bodies perturbing the free stream slightly

$$(M^2-1)\Phi_{\bar{x}\bar{x}}-\Phi_{\bar{y}\bar{y}}-\Phi_{\bar{z}\bar{z}}=0 \tag{1}$$

(the direction of the $\bar{x}$ axis of the coordinate system coupled to the body agrees with the free stream velocity direction at infinity). The coordinate transformation $\bar{x} = x_1\sqrt{M^2-1}$, $\bar{y} = y_1$, $\bar{z} = z_1$ reduces (1) to

$$\Phi_{x_1x_1}-\Phi_{y_1y_1}-\Phi_{z_1z_1}=0. \tag{2}$$

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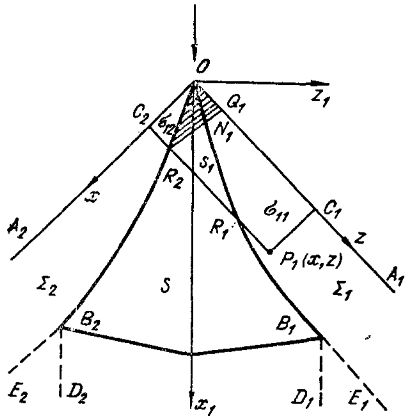

Fig. 1

The solution of the problem of the flow around a thin finite-span slightly-cambered wing when the conditions on the wing surface and on the vortical surface behind the wing are taken on the base plane $y_1 = 0$, is given by the formula [1]

$$\Phi(x_1, y_1, z_1) = -\frac{1}{\pi}\iint_T \Phi'_{\eta_1}(\xi_1, \zeta_1)\frac{d\xi_1\, d\zeta_1}{\sqrt{(x_1-\xi_1)^2-(z_1-\zeta_1)^2-y_1^2}}, \tag{3}$$

where T is the domain of integration on the base plane cut off by the characteristic cone with apex at the point (x_1, y_1, z_1) and $\Phi_{\eta_1}'(\xi_1, \zeta_1)$ is the normal derivative of the velocity potential on the base plane. The value of $\Phi_{\eta_1}'(\xi_1, \zeta_1)$ is not known in the part of the domain T outside the wing projection S on the base plane (Fig. 1) in the problem of the flow around a lifting surface. The potential equals zero of the base plane outside the wing projection and the wake behind it [1]

$$\Phi(x_1, 0, z_1) = 0, \ (x_1, z_1) \in \Sigma_i, \ i = 1, 2 \tag{4}$$

(Σ_i is the domain on the base plane bounded by the bow characteristic OA_i, the subsonic leading edge OB_i, the boundary of the vortical wake behind the wing B_iD_i). We shall later seek the solution in the part of the domain Σ_i where the influence of the vortical wake is not felt (in the domain bounded by the lines OA_i, OB_i and the characteristic line B_iE_i). The wing can be nonsymmetric with respect to the plane $z = 0$: $z = f(x)$, $x = f^-(z)$ is the equation of the leading edge OB_1 projection on the base plane, $z = \psi(x)$, $x = \psi^-(z)$ is the leading edge OB_2.

For the points $P_i(x, z) \in \Sigma_i$ $(i = 1, 2)$ condition (4) with (3) taken into account has the following form in the characteristic coordinate system $x = (x_1 - z_1)/\sqrt{2}$, $z = (x_1 + z_1)/\sqrt{2}$

$$\int_0^x\int_{f(\xi)}^{y} \frac{\theta_1(\xi,\zeta)\,d\zeta\,d\xi}{\sqrt{(x-\xi)(z-\zeta)}} + \int_0^x\int_{\psi(\xi)}^{f(\xi)} \frac{\alpha(\xi,\zeta)\,d\zeta\,d\xi}{\sqrt{(x-\xi)(z-\zeta)}} + \int_0^x\int_0^{\psi(\xi)} \frac{\theta_2(\xi,\zeta)\,d\zeta\,d\xi}{\sqrt{(x-\xi)(z-\zeta)}} = 0, \tag{5a}$$

$$\int_0^z\int_{\psi^-(\zeta)}^{x} \frac{\theta_2(\xi,\zeta)\,d\xi\,d\zeta}{\sqrt{(x-\xi)(z-\zeta)}} + \int_0^z\int_{f^-(\zeta)}^{\psi^-(\zeta)} \frac{\alpha(\xi,\zeta)\,d\xi\,d\zeta}{\sqrt{(x-\xi)(z-\zeta)}} + \int_0^z\int_0^{f^-(\zeta)} \frac{\theta_1(\xi,\zeta)\,d\xi\,d\zeta}{\sqrt{(x-\xi)(z-\zeta)}} = 0, \tag{5b}$$

where $\theta_i(\xi, \zeta) = \Phi_\eta'(\xi, \zeta)$ is a quantity unknown in the domain Σ_i, and $\alpha(\xi, \zeta) = \Phi_\eta'(\xi, \zeta)$ is a function given by the lifting surface geometry in the domain S. The domain of the dependence $T = \sigma_{11} + s_1 + \sigma_{12}$ is shown in Fig. 1 for the point $P_1(x, z) \in \Sigma_1$. The domains σ_{11}, s_1, σ_{12} correspond to the domain of integration of the first, second, and third terms of the relationship (5a): the domain σ_{11} is bounded by the lines R_1O, OC_1, C_1P_1, P_1R_1, the domain s_1 by R_1R_2, R_2O, OR_1, and the domain σ_{12} by R_2C_2, C_2O, OR_2. Let us rewrite the relationships (5a) and (5b) in the form

$$A_1(\theta_1) + F_1(\alpha) + B_{12}(\theta_2) = 0, \tag{6a}$$

$$A_2(\theta_2) + F_2(\alpha) + B_{21}(\theta_1) = 0. \tag{6b}$$

Here $A_i(\theta_i)$, $F_i(\alpha)$, $B_{ij}(\theta_j)$ are the first, second, and third terms of (5a) and (5b).

The initial operator $A_i(\theta_i)$ is a two-dimensional Abel operator in the quadrangular domain σ_{ii} bounded by projections of the leading edges, the bow characteristic, and the characteristic lines of the cone of the dependence of the point $P_i \in \Sigma_i$. The inversion formula for such an operator. $A_i^{-1}A_i(\theta_i) = \theta_i$, is presented in [5, pp. 174, 175]. Applying the operator A_1^{-1} to (6a), we obtain

$$\theta_1(x, z) + \frac{1}{\pi\sqrt{z - f(x)}}\left[\int\limits_{\psi(x)}^{f(x)} \frac{\alpha(x, \zeta)\sqrt{f(x) - \zeta}}{z - \zeta}\, d\zeta + \int\limits_0^{\psi(x)} \frac{\theta_2(x, \zeta)\sqrt{f(x) - \zeta}}{z - \zeta}\, d\zeta\right] = 0. \tag{7}$$

The value of $\theta_1(x, z)$ for the point $(x, z) \in \Sigma_1$ is determined in terms of $\Phi_\eta'(x, \zeta)$ on the characteristic $\xi = x$. On the section of this characteristic passing through the projection of the wing, on the line $R_2R_1(\psi(x) \le \zeta \le f(x))$, the value $\Phi_\eta'(x, \zeta) = \alpha(x, \zeta)$ is given by the wing geometry. On the section of the characteristic C_2R_2 $(0 \le \zeta \le \psi(x))$ in the domain σ_{12} the value $\Phi_\eta'(x, \zeta) = \theta_2(x, \zeta)$ is an unknown.

If the operator A_2^{-1} is applied to (6b), then we have for the point $(\xi, \zeta) \in \Sigma_2$

$$\theta_2(\xi, \zeta) + \frac{1}{\pi\sqrt{\xi - \psi^-(\zeta)}}\left[\int\limits_{f^-(\zeta)}^{\psi^-(\zeta)} \frac{\alpha(\xi', \zeta)\sqrt{\psi^-(\zeta) - \xi'}}{\xi - \xi'}\, d\xi' + \int\limits_0^{f^-(\zeta)} \frac{\theta_1(\xi', \zeta)\sqrt{\psi^-(\zeta) - \xi'}}{\xi - \xi'}\, d\xi'\right] = 0, \tag{8}$$

from which $\theta_2(x, \zeta)$ can be found on the characteristic $\xi = x$:

$$\theta_2(x, \zeta) = \frac{-1}{\pi\sqrt{x - \psi^-(\zeta)}}\left[\int\limits_{f^-(\zeta)}^{\psi^-(\zeta)} \frac{\alpha(\xi', \zeta)\sqrt{\psi^-(\zeta) - \xi'}}{x - \xi'}\, d\xi' + \int\limits_0^{f^-(\zeta)} \frac{\theta_1(\xi', \zeta)\sqrt{\psi^-(\zeta) - \xi'}}{x - \xi'}\, d\xi'\right]. \tag{9}$$

Substituting $\theta_2(x, \zeta)$ from (9) into (7), we obtain a two-dimensional Volterra-type integral equation of the second kind in the function $\theta_1(x, z)$:

$$\begin{aligned}\theta_1(x, z) = \frac{1}{\pi\sqrt{z - f(x)}}\Bigg\{&-\int\limits_{\psi(x)}^{f(x)} \frac{\alpha(x, \zeta)\sqrt{f(x) - \zeta}}{z - \zeta}\, d\zeta + \\ &+\frac{1}{\pi}\int\limits_0^{\psi(x)}\int\limits_{f^-(\zeta)}^{\psi^-(\zeta)} \frac{\alpha(\xi, \zeta)\sqrt{[f(x) - \zeta][\psi^-(\zeta) - \xi]}}{(x - \xi)(z - \zeta)\sqrt{x - \psi^-(\zeta)}}\, d\xi\, d\zeta + \\ &+\frac{1}{\pi}\int\limits_0^{\psi(x)}\int\limits_0^{f^-(\zeta)} \frac{\theta_1(\xi, \zeta)\sqrt{[f(x) - \zeta][\psi^-(\zeta) - \xi]}}{(x - \xi)(z - \zeta)\sqrt{x - \psi^-(\zeta)}}\, d\xi\, d\zeta\Bigg\}.\end{aligned} \tag{10}$$

The value of $\theta_1(x, z)$ at the point $P_1(x, z) \in \Sigma_1$ is determined in terms of known values of $\alpha(\xi, \zeta)$ on the characteristic $R_1R_2(\psi(x) \le \zeta \le f(x))$ and on the nose part of the domain of dependence on the wing s_1 (the domain bounded by the lines R_2O, ON_1, N_1R_2) and in terms of the unknown value $\theta_1(\xi, \zeta)$ in the nose part of the domain Σ_1 (the domain bounded by the lines N_1O, OQ_1, Q_1N_1) (see Fig. 1). A relationship analogous to (10) can be written down for $Q_2(x, z)$ at the point $P_2(x, z) \in \Sigma_2$.

The normal component of the velocity $\theta_1(x, z)$ in the domain Σ_1 has a singularity of the type $r^{-1/2}$ approaching the leading edge $(z \to f(x))$. We write (10) as

$$\tau_1 = G_1\alpha + I_1\tau_1,\ \tau_1(x, z) = \theta_1(x, z)\sqrt{z - f(x)}. \tag{11}$$

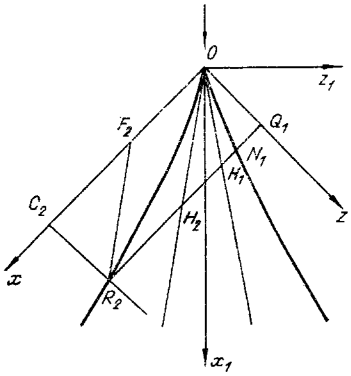

Fig. 2

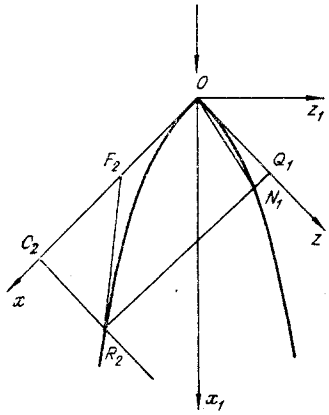

Fig. 3

The function

$$G_1\alpha = -\frac{1}{\pi}\int\limits_{\psi(x)}^{f(x)} \frac{\alpha(x,\zeta)\sqrt{f(x)-\zeta}}{z-\zeta}\,d\zeta + \frac{1}{\pi^2}\int\limits_0^{\psi(x)}\int\limits_{f^-(\zeta)}^{\psi^-(\zeta)} \frac{\alpha(\xi,\zeta)\sqrt{[f(x)-\zeta][\psi^-(\zeta)-\xi]}}{(x-\xi)(z-\zeta)\sqrt{x-\psi^-(\zeta)}}\,d\xi\,d\zeta$$

is expressed in terms of values $\Phi_\eta{}' = \alpha$ known on the wing surface and belongs to the class of continuous functions for smooth wings.

Let us estimate the norm of the operator

$$I_1\tau_1 = \frac{1}{\pi^2}\int\limits_0^{\psi(x)}\int\limits_0^{f^-(\zeta)} \frac{\tau_1(\xi,\zeta)\sqrt{[f(x)-\zeta][\psi^-(\zeta)-\xi]}}{(x-\xi)(z-\zeta)\sqrt{[x-\psi^-(\zeta)][\zeta-f(\xi)]}}\,d\xi\,d\zeta,$$

in which integration is performed over the curvilinear triangle N_1OR_1 in the nose part of the domain Σ_1

$$\|I_1\| = \frac{1}{\pi}\int\limits_0^{\psi(x)}\left|\frac{\sqrt{f(x)-\zeta}}{(z-\zeta)\sqrt{x-\psi^-(\zeta)}}H(x,\zeta)\right|d\zeta,$$

$$H(x,\zeta) = \frac{1}{\pi}\int\limits_0^{f^-(\zeta)} \frac{\sqrt{\psi^-(\zeta)-\xi}}{(x-\xi)\sqrt{\zeta-f(\xi)}}\,d\xi \leqslant \frac{1}{\pi}\sqrt{\psi^-(\zeta)}\int\limits_0^{f^-(\zeta)} \frac{d\xi}{(x-\xi)\sqrt{\zeta-f(\xi)}}.$$

Let us note that a sign-constant positive function is under the integral of the operator I_1 (including the operator H). The sign-positivity of the integrands and their integrals is not spoiled during changes in the limits of integration made for carrying out the estimates.

The cases of concave (Fig. 2) and convex (Fig. 3) leading edges should be distinguished when performing the estimates. A fragment of the nose part of the wing is given in Figs. 2 and 3 (see Fig. 1). For concave wings we draw the tangent to the leading edge OB_1 at the wing apex (the line OH_1) whose equation is $\zeta = k_{12}\xi$ $(1 \le k_{12} \le \infty)$. For convex wings we connect the wing apex with the point of intersection N_1 of the characteristic R_2Q_1 with the leading edge by a straight line [the equation of the line ON_1 is $\zeta = k_{13}\xi$ $(1 \le k_{13} \le \infty)$]. In both cases $\zeta = f(\xi) < k_1\xi$ in the interval under consideration $0 \le \zeta \le \psi(x)$ (the coordinates of the point $R_2[x, \psi(x)]$). Taking this into account

$$H \leqslant \frac{1}{\pi}\sqrt{\psi^-(\zeta)}\int\limits_0^{\zeta/k_1} \frac{d\xi}{(x-\xi)\sqrt{\zeta-k_1\xi}} =$$

$$= \frac{1}{\pi}\sqrt{\psi^-(\zeta)}\frac{2}{\sqrt{k_1x-\zeta}}\arctan\left.\sqrt{\frac{\zeta-k_1\xi}{k_1x-\zeta}}\right|_0^{\zeta/k_1} \leqslant \sqrt{\frac{\psi^-(\zeta)}{k_1x-\zeta}}.$$

TABLE 1

$\frac{\operatorname{tg}\mu}{\operatorname{tg}\varkappa}$	$k=5{,}671,\ \chi=55$		$k=2{,}145,\ \chi=70$		$k=1{,}428,\ \chi=80$	
	θ_{10}	θ_1	θ_{19}	θ_1	θ_{16}	θ_1
1,05	1,923805	1,924042	1,541694	1,552448	0,970179	1,015902
1,1	1,165466	1,165626	0,909604	0,916793	0,535812	0,565723
1,2	0,663620	0,663723	0,500043	0,504623	0,270223	0,288625
4/3	0,416764	0,416836	0,304412	0,307549	0,152776	0,164930
2	0,129256	0,129283	0,087831	0,088978	0,037555	0,041602
8/3	0,069286	0,069302	0,045713	0,046363	0,018403	0,020607
3	0,054871	0,054884	0,035872	0,036393	0,014182	0,015935
4	0,032104	0,032112	0,020624	0,020939	0,007883	0,008914
6	0,015894	0,015898	0,010043	0,010203	0,003722	0,004235
12	0,005152	0,005153	0,003205	0,003258	0,001154	0,001321

Therefore

$$\|I_1\|\leqslant\frac{1}{\pi}\int\limits_0^{\psi(x)}\frac{\sqrt{f(x)-\zeta}}{(z-\zeta)\sqrt{x-\psi^-(\zeta)}}\sqrt{\frac{\psi^-(\zeta)}{k_1x-\zeta}}\,d\zeta,$$

$$\|I_1\|\leqslant\frac{1}{\pi}\int\limits_0^{\psi(x)}\frac{\sqrt{k_1x-\zeta}}{(z-\zeta)\sqrt{x-\psi^-(\zeta)}}\sqrt{\frac{\psi^-(\zeta)}{k_1x-\zeta}}\,d\zeta=\frac{1}{\pi}\int\limits_0^{\psi(x)}\frac{\sqrt{\psi^-(\zeta)}\,d\zeta}{(z-\zeta)\sqrt{x-\psi^-(\zeta)}}.$$

The cases of concave (Fig. 2) and convex (Fig. 3) wings should again be differentiated in further execution of the estimates. For concave edges we draw a line R_2F_2 from the point $R_2[x, \psi(x)]$, whose equation is $\xi = g_{22}(\zeta) = x - [\psi(x) - \zeta]/k_{22}$, $R_2F_2 \| OH_2$ (k_{22} is the tangent of the slope of the tangent to the leading edge OB_2 at the wing apex). For convex wings, we draw the line R_2F_2 from the point $R_2[x, \psi(x)]$, whose equation is $\xi = g_{23}(\zeta) = x - [\psi(x) - \zeta]/k_{23}$ [k_{23} is the tangent of the slope of the tangent to the leading edge OB_2 at the point R_2 $(0 \le k_{22}, k_{23} \le 1)$]. In both cases $\xi = \psi^-(\zeta) \le g_2(\zeta)$ in the interval under consideration $0 \le \zeta \le \psi(x)$ and therefore

$$\|I_1\|\leqslant\frac{1}{\pi}\int\limits_0^{\psi(x)}\frac{\sqrt{g_2(\zeta)}}{(z-\zeta)\sqrt{x-g_2(\zeta)}}\,d\zeta.$$

Taking into account that $g_2(\zeta) \le x$, $x - g_2(\zeta) = [\psi(x) - \zeta]/k_2$, we write

$$\|I_1\|\leqslant\frac{\sqrt{k_2x}}{\pi}\int\limits_0^{\psi(x)}\frac{d\zeta}{(z-\zeta)\sqrt{\psi(x)-\zeta}}=\frac{\sqrt{k_2x}}{\pi}\frac{2}{\sqrt{z-\psi(x)}}\operatorname{arctg}\sqrt{\frac{\psi(x)-\zeta}{z-\psi(x)}}\Bigg|_0^{\psi(x)}\leqslant\sqrt{\frac{k_2x}{z-\psi(x)}}.$$

In the domain Σ_1 $f(x) \le z \le \infty$, and we finally have the estimate

$$\|I_1\|\leqslant\sqrt{\frac{k_2x}{f(x)-\psi(x)}},\tag{12}$$

where $0 \le k_2 \le 1$ is the maximal value of the tangent of the slope of the leading edge OB_2 in the section $0 \le \xi \le x$ ($P_1(x, z) \in \Sigma_1$) for both the concave and convex edge OB_2 cases.

The necessary condition for convergence of the solution of (11) by successive approximations is the requirement $||I_1|| \le 1$. If $||I_1|| < 1$, then the series

$$\tau_1=\sum_{n=0}^{\infty}I_1^n(G_1\alpha)\tag{13}$$

will be the unique solution of (11) [6, pp. 126 and 127].

Let us examine the limits of applicability of the successive approximation for the upper bounds found for wings whose leading edge projections on the base plane are straight lines ($z = k_ix$ is the equation of the line OB_i). According to (12) the necessary condition for convergence in this case has the form

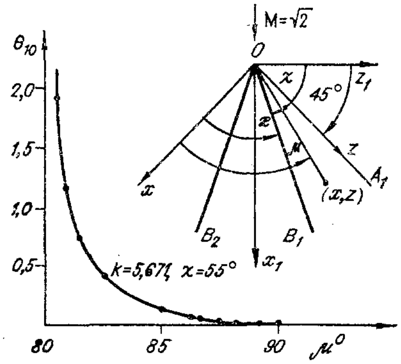

Fig. 4

$$\|I_1\| \leqslant \sqrt{\frac{k_2}{k_1 - k_2}} < 1. \tag{14}$$

For a wing symmetric relative to the z = 0 plane ($k_2 = 1/k_1 = 1/k$, $k \geq 1$), it follows from (14) that $k > \sqrt{2}$. This corresponds to sweepback angles of $45° \leq \chi \leq 81°$ ($\chi = 45°$ is the sonic edge). For a wing whose leading edge OB_1 is directed along the stream ($k_1 = 1$, $\chi_1 = 90°$), $k_2 > 1/2$ which corresponds to sweepback angles of $45° \leq \chi_2 \leq 72°$. Analogously for a wing whose leading edge OB_2 is directed along the stream ($k_2 = 1$, $\chi_2 = 90°$), $k_1 > 2$ (sweepback angles of $45° \leq \chi_1 \leq 72°$). In all three examples the angle at the triangle apex is $\approx 18°$. Let us recall that k_i are tangents of the slope of the leading edges determined in the characteristic coordinate system Oxz (k = 0 is the Ox axis and $k = \infty$ is the Oz axis), while the sweepback angles are, as usual, measured from the direction of the Oz_1 axis of the initial Ox_1z_1 coordinate system (see Fig. 1).

The zeroth approximation of the solution of (11) is the first term $\tau_{10}(x, z) = G_1\alpha$ of the series (13). According to (11), the zeroth approximation of the derivative of the velocity potential on the base plane outside the wing is

$$\theta_{10}(x, z) = \frac{G_1\alpha}{\sqrt{z - f(x)}}, \quad (x, z) \in \Sigma_1. \tag{15}$$

The θ_{10} are evaluated according to (15) in the case of the flow around the plane triangular plates, symmetric with respect to the z = 0 plane, with different sweepback angles. Results of the computations of θ_{10} on rays μ passing through the point $(x, z) \in \Sigma_1$ are presented in the table for three plates, k = 5.671, $\chi = 55°$; k = 2.145, $\chi = 70°$; and k = 1.428, $\chi = 80°$. The ray angle μ (right side of Fig. 4) is given in terms of the value of the tangent of the leading edge slope $\tan\mu = ak$ (a = 1.05, 1.1, ..., 6, 12). The ray $\tan\mu = k$ corresponds to the leading edge OB_1 ($k = \tan\varkappa$), the ray $\tan\mu = \infty$ to the bow characteristic OA_1. A graph of $\theta_{10} = \theta_{10}(\mu)$ is given in the left side of Fig. 4 for the plate k = 5.671, $\chi = 55°$ ($\mu = 80°$ corresponds to the leading edge OB_1 and $\mu = 90°$ to the bow characteristic OA_1).

Also presented in Table 1 are values of θ_1 on these same rays according to the exact solution [7, p. 129]. The values of θ_{10} and θ_1 have the $r^{-1/2}$ singularity on the leading edge OB_1, and $\theta_{10} = \theta_1 = 0$ on the characteristic OA_1. On the ray nearest to the leading edge ($\tan\mu = 1.05k$), the deviation of the values of θ_{10} from the exact solution is less than 0.01% for the plate k = 5.671, $\chi = 55°$, less than 1% for k = 2.145, $\chi = 70°$, and less than 5% for k = 1.428, $\chi = 80°$.

As should have been expected, the deviation of the zeroth approximation from the exact solution increases as the degree of leading edge standoff from the sonic increases. But even in the case of the narrowest wing used in aircraft construction practice $\chi = 80°$, a 5% error for the number $M = \sqrt{2}$ in the determination of the downwash behind a wing during computation of the total aerodynamic characteristics will be less significant. As the Mach number increases ($M > \sqrt{2}$) the degree of leading edge standoff from the sonic of a specific wing diminishes with a given value of the sweepback angle χ, and computation of the wing aerodynamic characteristics by the zeroth approximation becomes more reliable.

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SELF MOTION OF A BODY IN A FLUID

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UDC 532.516

1. Many bodies (ships, living creatures) are capable of self-motion in a fluid, i.e., they move themselves by pushing fluid away from them.

The well-known (see [1], for example) equations of motion of a rigid body with respect to an inertial reference frame are

$$\frac{d\mathbf{P}_b}{dt} = \mathbf{F};\ \frac{d\mathbf{L}_b}{dt} = \mathbf{N},$$

where t is the time, $\mathbf{P}_b$ is the momentum of the body, $\mathbf{F}$ is the total external force acting on the body, $\mathbf{L}_b$ is the angular momentum of the body about the point O (the origin of the coordinate system), $\mathbf{N}$ is the total external torque acting on the body about point O. Therefore in the case when the body translates by pushing away the surrounding fluid we must have

$$\frac{d\mathbf{P}_b}{dt} = \mathbf{S}_{f\to b} \tag{1.1}$$

$$\frac{d\mathbf{L}_b}{dt} = \mathbf{T}_{f\to b} \tag{1.2}$$

where $\mathbf{S}_{f\to b}$ is the momentum transferred by the fluid to the body per unit time and $\mathbf{T}_{f\to b}$ is the angular momentum transferred by the fluid to the body per unit time about point O. Equations (1.1) and (1.2) are the basic equations describing self-motion of a body in a fluid.

In the presence of body forces, the total force acting on the body must be added to the right hand side of (1.1) and the total moment of the forces (torque) about point O must be added to the right hand side of (1.2).

Self-motion of a body in a fluid is possible because of the interaction between the boundary of the body and the fluid (but not as the result of any disturbances in the fluid which could also occur in the absence of the body). Hence the boundary of the self-moving body serves as its driver. In self-motion the operation of the driver is such that conditions exist on the boundary of the body for which the equations of self-motion are satisfied.

2. An approximate solution of the problem was found in [2, 3] for steady flow of a viscous incompressible fluid past a self-moving body (a circular cylinder and a sphere). In the

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